"standard"

4.5 Amplitude modulation: AM

4.53. DSB-SC amplitude modulation (which is summarized in Figure 18) is easy to understand and analyze in both time and frequency domains. However, analytical simplicity is not always accompanied by an equivalent simplicity in practical implementation.

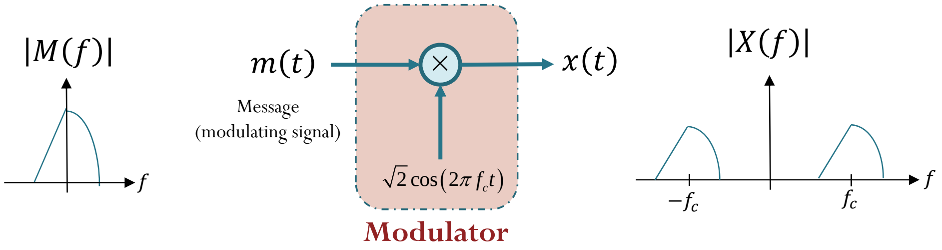


Figure 18: DSB-SC modulation.

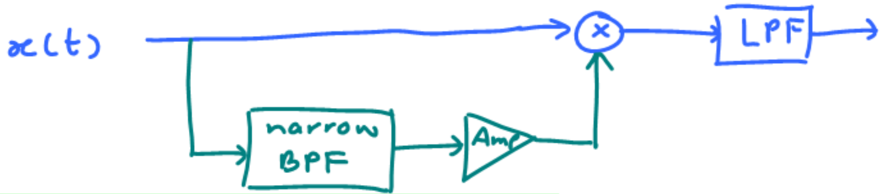
Problem: The (coherent) demodulation of DSB-SC signal requires the receiver to possess a carrier signal that is synchronized with the incoming carrier. This requirement is not easy to achieve in practice because the modulated signal may have traveled hundreds of miles and could even suffer from some unknown frequency shift.

4.54. If a carrier component is transmitted along with the DSB signal, demodulation can be simplified.

$$x(t) = m(t)\cos(2\pi f_c t) + A\cos(2\pi f_c t)$$

(a) The received carrier component can be extracted using a narrowband bandpass filter and can be used as the demodulation carrier. (There is no need to generate a carrier at the receiver.)

At receiver



(b) If the carrier amplitude is sufficiently large, the need for generating a demodulation carrier can be completely avoided.

- This will be the focus of this section.

AM

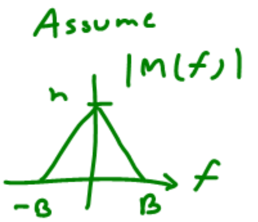
Definition 4.55. For AM, the transmitted signal is typically defined as

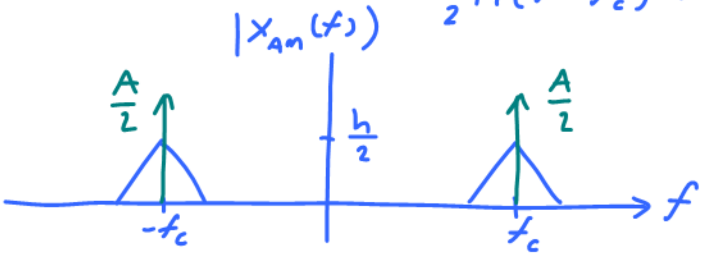
$$x_{\mathrm{AM}}(t) = (A + m(t))\cos(2\pi f_c t) = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t)\cos(2\pi f_c t)}_{\text{sidebands}}$$

4.56. Spectrum of $x_{\mathrm{AM}}(t)$:

$\frac{A}{2}\delta(f-f_c)+\frac{A}{2}\delta(f+f_c)$

$\frac{1}{2}M(f-f_c)+\frac{1}{2}M(f+f_c)$





- Basically the same as that of DSB-SC signal except for the two additional impulses (**discrete** spectral component) at the carrier frequency $\pm f_c$.
    - This is why we say the DSB-SC system is a ***suppressed carrier*** system.

Definition 4.57. Consider a signal $A(t)\cos(2\pi f_c t)$. If $A(t)$ varies slowly in comparison with the sinusoidal carrier $\cos(2\pi f_c t)$, then the ***envelope*** $E(t)$ of $A(t)\cos(2\pi f_c t)$ is $|A(t)|$.

$x_{AM}(t) = A\cos(2\pi f_c t) + m(t)\cos(2\pi f_c t)$

$= \underbrace{(A+m(t))}_{A(t)}\cos 2\pi f_c t$

4.58. ***Envelope of AM signal***: For AM signal, $A(t) = A + m(t)$ and

$$E(t) = |A + m(t)|.$$

See Figure 19.

case (a) If $\forall t$, $A(t) > 0$, then $E(t) = A(t) = A + m(t)$

- The envelope has the same shape as $m(t)$.
- We can detect the desired signal $m(t)$ by detecting the envelope (envelope detection).

case (b) If $\exists t$, $A(t) < 0$, then $E(t) \neq A(t)$.

- The envelope shape differs from the shape of $m(t)$ because the negative part of $A + m(t)$ is rectified.
    - This is referred to as **phase reversal** and envelope distortion.

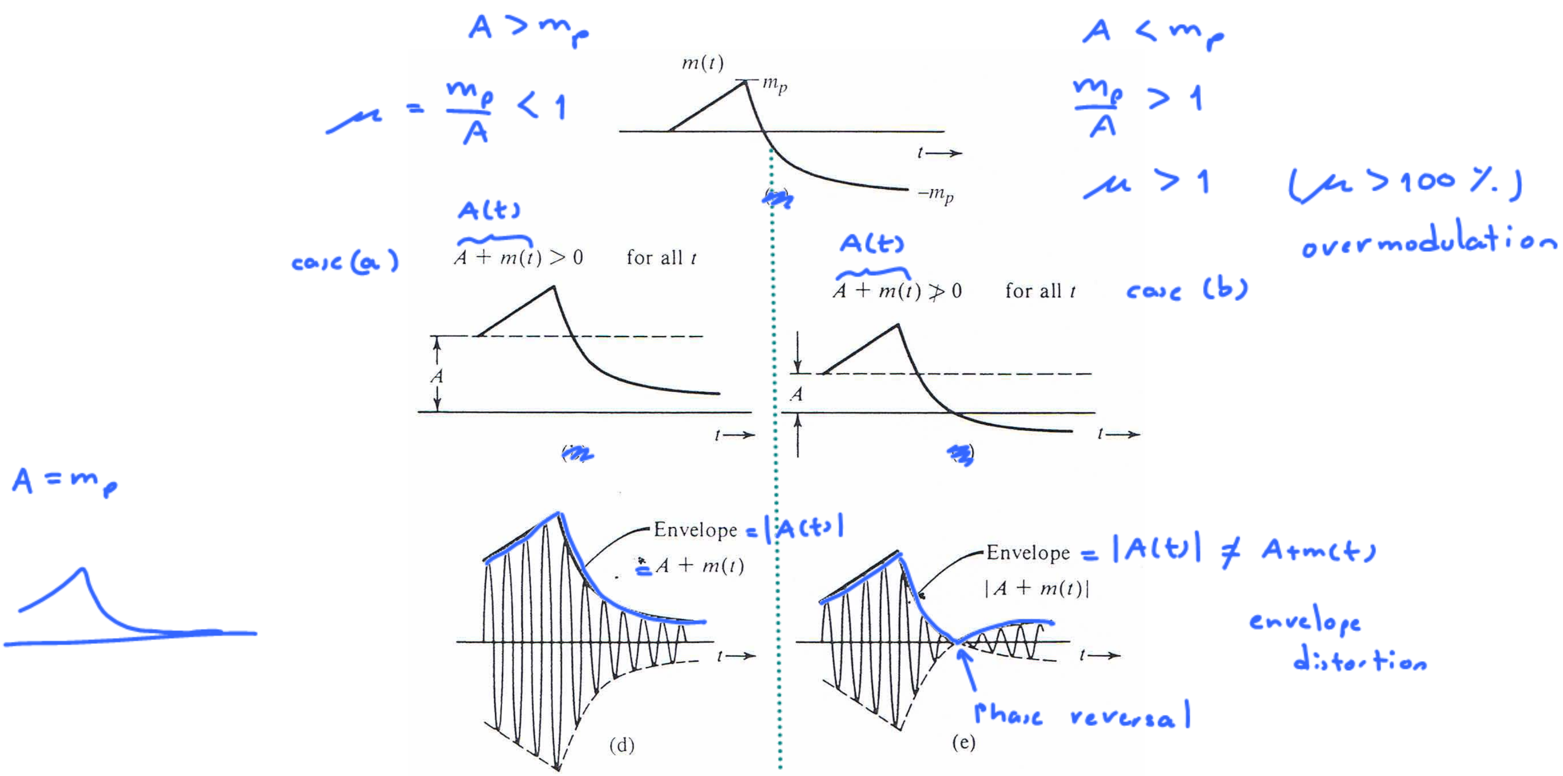


Figure 19: AM signal and its envelope [5, Fig 4.8]

Definition 4.59. The positive constant

$$\mu \equiv \frac{\max\limits_t \text{(envelope of the sidebands)}}{\max\limits_t \text{(envelope of the carrier)}} = \frac{\max\limits_t |m(t)|}{\max\limits_t |A|} = \frac{m_p}{A}$$

is called the **modulation index**.

- $m_p \equiv \max\limits_t |m(t)|$
    - By the way m_p is defined, the message $m(t)$ is between $\pm m_p$.
- The quantity $\mu \times 100\%$ is often referred to as the percent modulation.

Example 4.60. Consider a sinusoidal (pure-tone) message $m(t) = A_m \cos(2\pi f_m t)$. Suppose $A = 1$. Then, $\mu = A_m$. Figure 20 shows the effect of changing the value modulation index on the modulated signal.

$\mu = \frac{m_p}{A} = \frac{1}{2}$

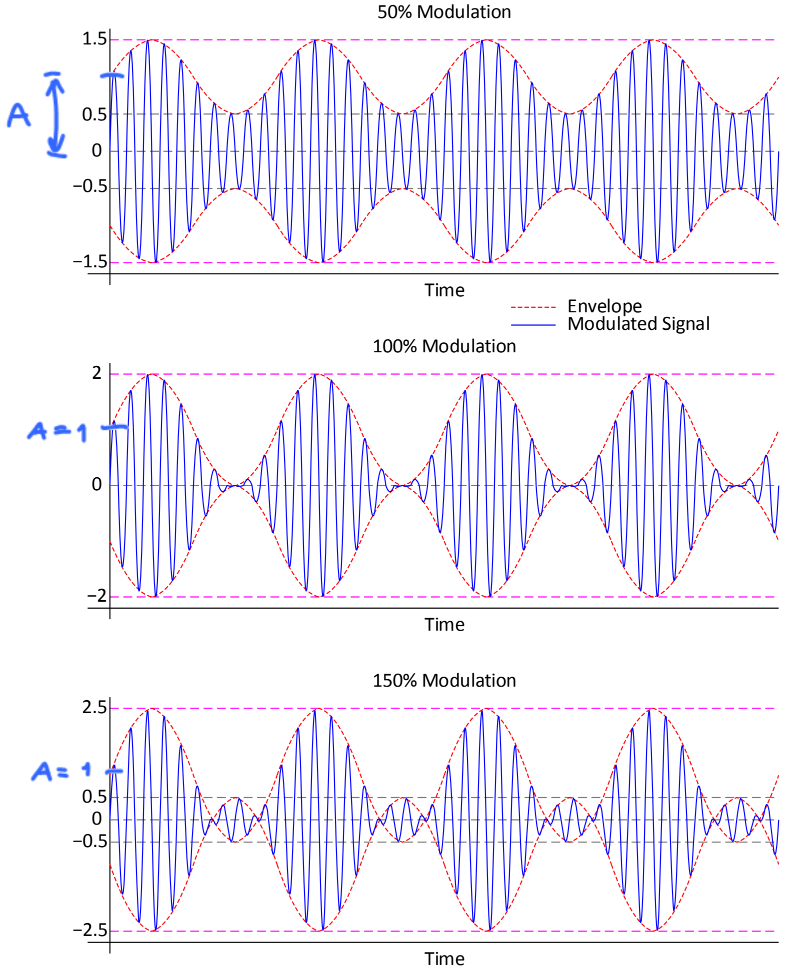


Figure 20: Modulated signal in standard AM with sinusoidal message

4.61. It should be noted that the ratio that defines the modulation index compares the maximum of the two envelopes. In other references, the notation for the AM signal may be different but the idea (and the corresponding motivation) that defines the modulation index remains the same.

- In [3, p 163], it is assumed that $m(t)$ is already scaled or normalized to have a magnitude not exceeding unity ($|m(t)| \leq 1$) [3, p 163]. There,

$$x_{\mathrm{AM}}(t) = A_c (1 + \mu m(t)) \cos(2\pi f_c t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{A_c \mu m(t) \cos(2\pi f_c t)}_{\text{sidebands}}.$$

  - $m_p = 1$
  - The modulation index is then

$$\frac{\max_t(\text{envelope of the sidebands})}{\max_t(\text{envelope of the carrier})} = \frac{\max_t |A_c\mu m(t)|}{\max_t |A_c|} = \frac{|A_c\mu|}{|A_c|} = \mu.$$

- In [14, p 116],

$$x_{\text{AM}}(t) = A_c\left(1+\mu\frac{m(t)}{m_p}\right)\cos(2\pi f_c t) = \underbrace{A_c\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{A_c\mu\frac{m(t)}{m_p}\cos(2\pi f_c t)}_{\text{sidebands}}.$$

  - The modulation index is then

$$\frac{\max_t(\text{envelope of the sidebands})}{\max_t(\text{envelope of the carrier})} = \frac{\max_t \left|A_c\mu\frac{m(t)}{m_p}\right|}{\max_t |A_c|} = \frac{|A_c|\,\mu\frac{m_p}{m_p}}{|A_c|} = \mu.$$

4.62. Power of the transmitted signals.

(a) In DSB-SC system, recall, from 4.52, that, when

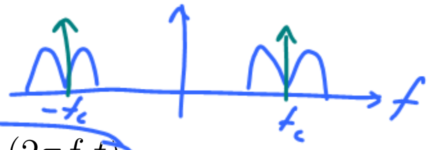
$$x(t) = m(t)\cos(2\pi f_c t)$$

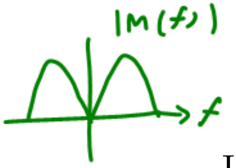
with f_c sufficiently large, we have

$$P_x = \frac{1}{2}P_m.$$

Therefore, all transmitted power are in the sidebands which contain message information.

(b) In AM system,





$$x_{\text{AM}}(t) = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t)\cos(2\pi f_c t)}_{\text{sidebands}}$$

If we assume that the average of $m(t)$ is 0 (no DC component), then the spectrum of the sidebands $m(t)\cos(2\pi f_c t+\theta)$ and the carrier $A\cos(2\pi f_c t+\theta)$ are non-overlapping in the frequency domain. Hence, when f_c is sufficiently large

$$P_x = \frac{1}{2}A^2 + \frac{1}{2}P_m.$$

wasted (from the power eff. perspective)

useful part

Linear modulation — DSB

DSB — DSB - SC ← suppressed carrier

DSB — AM ← the carrier is not suppressed

$$\mu = \frac{m_p}{A} \Rightarrow A = \frac{m_p}{\mu}$$

$$\Downarrow$$

- Efficiency:

$$\frac{\text{useful power}}{\text{total power}} = \frac{\frac{P_m}{2}}{\frac{A^2}{2}+\frac{P_m}{2}} = \frac{P_m}{A^2+P_m} = \frac{1}{1+\frac{A^2}{P_m}} = \frac{1}{1+\frac{m_p^2}{\mu^2 P_m}}$$

- For high power efficiency, we want small $\frac{m_p^2}{\mu^2 P_m}$.
    - By definition, $|m(t)| \le m_p$. Therefore, $\frac{m_p^2}{P_m} \ge 1$. Smallest value is 1 which happens when $m(t)$ takes only two values: m_p and $-m_p$.

$$|m(t)|^2 \le m_p^2$$

$$P_m = \langle |m(t)|^2 \rangle \le m_p^2$$

    - Want μ to be large. However, when $\mu > 1$, we have phase reversal. So, the largest value of μ is 1.
    - The best power efficiency we can achieved is then 50%.

$$\frac{m_p^2}{\mu^2 P_m} \ge 1$$

the "best" value = 1

$$\text{eff.} = \frac{1}{1+1} = \frac{1}{2} = 50\%$$

- Conclusion: at least 50% (and often close to 2/3[3, p. 176]) of the total transmitted power resides in the carrier part which is independent of $m(t)$ and thus conveys no message information.

4.63. An AM signal can be demodulated using the same coherent demodulation technique that was used for DSB. However, the use of coherent demodulation negates the advantage of AM

- Note that, conceptually, the received AM signal is the same as DSB-SC signal except that the $m(t)$ in the DSB-SC signal is replaced by $A(t) = A + m(t)$. We also assume that A is large enough so that $A(t) \ge 0$.
- Recall the key equation of ***switching demodulator*** (45):

$$\text{LPF}\{\underbrace{A(t)\cos(2\pi f_c t)}_{\text{received signal}} \times 1[\cos(2\pi f_c t) \ge 0]\} = \frac{1}{\pi}A(t) \tag{48}$$

We noted before that this technique requires the switching to be in sync with the incoming cosine.

Even when we change to "switching operation", the ON-OFF pattern will still have to be in sync with the received signal.

These two sinusoids must be in sync.
- same phase
- same freq.
- same delay

In particular, the ON part must happen when the received cosine is positive. The OFF part must happen when the received cosine is negative.

New idea (via new circuit element)
(diode)

4.64. ***Demodulation*** of AM Signals via **rectifier detector**: The receiver will first recover $A + m(t)$ and then remove A.

$$x_{AM}(t) = (\underbrace{A+m(t)}_{\geq 0})\cos(2\pi f_c t) \rightarrow \boxed{\text{HWR}} \xrightarrow{v(t)} (A+m(t))\cos \times 1[\cos] \rightarrow \boxed{\text{LPF}} \rightarrow \frac{A+m(t)}{\pi}$$

- When $\forall t, A(t) \geq 0$, we can replace the switching demodulator by the ***rectifier demodulator/detector***. In which case, we suppress the negative part of $A(t)\cos(2\pi f_c t)$ using a diode (half-wave rectifier: HWR).
    - Here, we define a HWR to be a memoryless device whose input-output relationship is described by a function $f_{\text{HWR}}(\cdot)$:

$$f_{\text{HWR}}(x) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

$$y(t) = f_{HWR}(x(t)) = \begin{cases} x(t), & x(t) \geq 0, \\ 0, & x(t) < 0 \end{cases}$$

- This is mathematically equivalent to a switching demodulator in (45) and (48).

$$v(t) = f_{HWR}(x_{AM}(t)) = f_{HWR}((A+m(t))\cos(2\pi f_c t))$$

$$= \begin{cases} (A+m(t))\cos(2\pi f_c t), & (\overset{\geq 0}{\cancel{A+m(t)}})\cos(2\pi f_c t) \geq 0, \\ 0, & (\overset{\geq 0}{\cancel{A+m(t)}})\cos(2\pi f_c t) < 0. \end{cases}$$

$$= (A+m(t))\cos(2\pi f_c t) \times \boxed{\begin{cases} 1, & \cos(2\pi f_c t) \geq 0, \\ 0, & \cos(2\pi f_c t) < 0 \end{cases}} \leftarrow 1[\cos(2\pi f_c t) \geq 0]$$

- It is in effect synchronous detection performed without using a local carrier [4, p 167].
- This method needs $A(t) \geq 0$ so that the sign of $A(t)\cos(2\pi f_c t)$ will be the same as the sign of $\cos(2\pi f_c t)$.
- The dc term $\frac{A}{\pi}$ may be blocked by a capacitor to give the desired output $m(t)/\pi$.

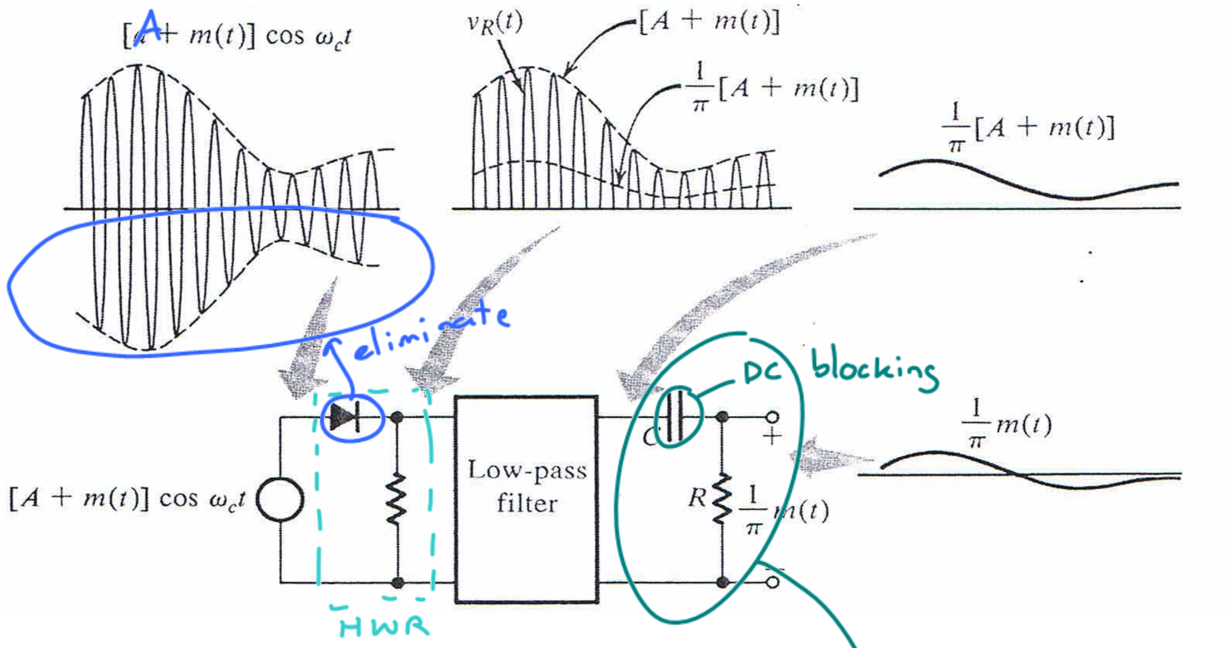


Figure 21: Rectifier detector for AM [5, Fig. 4.10].

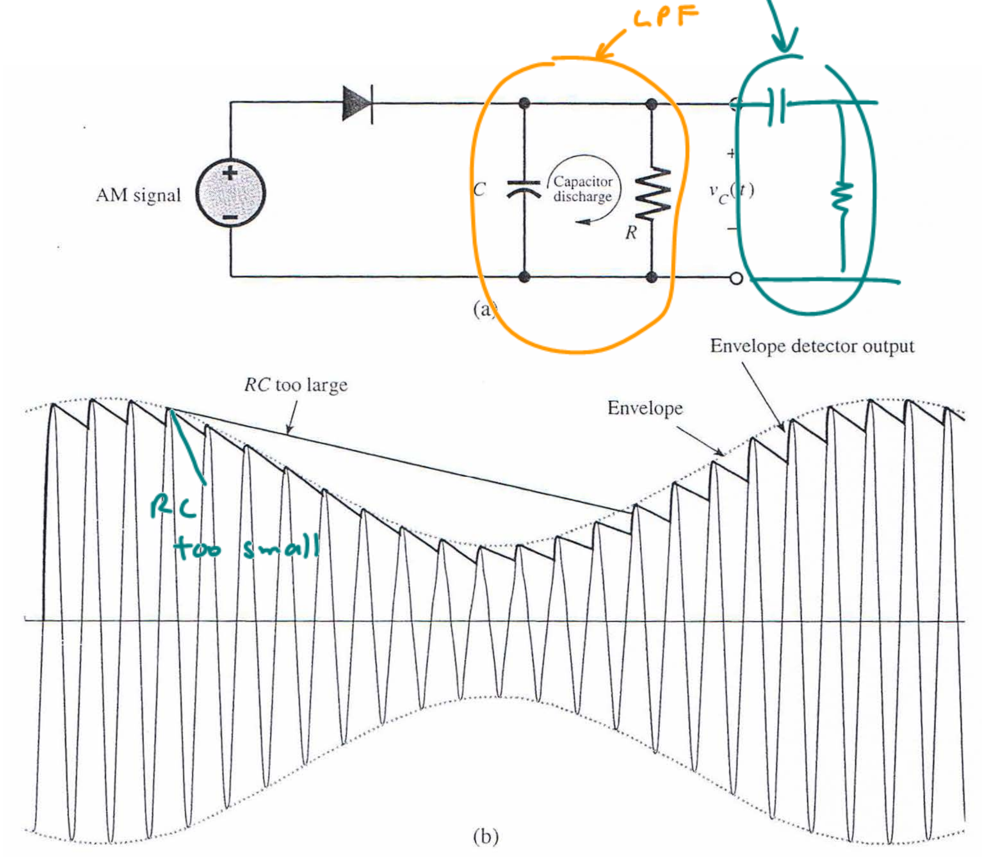


Figure 22: Envelope detector for AM [5, Fig. 4.11].

4.65. Demodulation of AM signal via ***envelope detector***:

- Design criterion of RC:

$$2\pi B \ll \frac{1}{RC} \ll 2\pi f_c.$$

- The envelope detector output is $A + m(t)$ with a ripple of frequency f_c.
- The dc term can be blocked out by a capacitor or a simple RC high-pass filter.
- The ripple may be reduced further by another (low-pass) RC filter.

4.66. AM Trade-offs:

(a) *Disadvantages*:

- Higher power and hence higher cost required at the transmitter
- The carrier component is wasted power as far as information transfer is concerned.
- Bad for power-limited applications.

(b) *Advantages*:

synchronization is automatic
easy

- Coherent reference is not needed for demodulation.
- Demodulator (receiver) becomes simple and inexpensive.
- For broadcast system such as commercial radio (with a huge number of receivers for each transmitter),
  - any cost saving at the receiver is multiplied by the number of receiver units.
  - it is more economical to have one expensive high-power transmitter and simpler, less expensive receivers.

(c) Conclusion: Broadcasting systems tend to favor the trade-off by migrating cost from the (many) receivers to the (fewer) transmitters.

4.67. References: [3, p 198–199], [5, Section 4.3] and [13, Section 3.1.2].